RECOGNISING ACHIEVEMENT

ADVANCED GCE

4729/01

MATHEMATICS

Mechanics 2

WEDNESDAY 9 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

* Write your name, centre number and candidate number in the spaces provided on the answer booklet.

* Read each question carefully and make sure you know what you have to do before starting your answer.

* Answer **all** the questions.

* Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

* The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

* You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

* The number of marks is given in brackets [ ] at the end of each question or part question.

* The total number of marks for this paper is 72.

* **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

OCR is an exempt Charity   **[Turn over**

1   A ball is projected with speed $12\,\text{m s}^{-1}$ at an angle of elevation of 55° above the horizontal. At the instant when the ball reaches its greatest height, it hits a vertical wall, which is perpendicular to the ball's path. The coefficient of restitution between the ball and the wall is 0.65. Calculate the speed of the ball

   (i) immediately before its impact with the wall,   [2]

   (ii) immediately after its impact with the wall.   [2]

2   A particle of mass $m\,\text{kg}$ is projected directly up a rough plane with a speed of $5\,\text{m s}^{-1}$. The plane makes an angle of 30° with the horizontal and the coefficient of friction is 0.2. Calculate the distance the particle travels up the plane before coming instantaneously to rest.   [6]

3
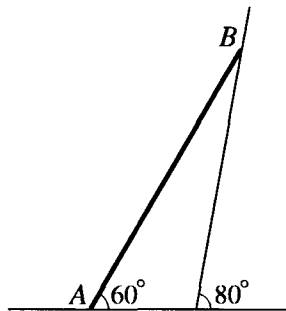
A uniform rod AB, of weight 25 N and length 1.6 m, rests in equilibrium in a vertical plane with the end A in contact with rough horizontal ground and the end B resting against a smooth wall which is inclined at 80° to the horizontal. The rod is inclined at 60° to the horizontal (see diagram). Calculate the magnitude of the force acting on the rod at B.   [6]

4   A car of mass 1200 kg has a maximum speed of $30\,\text{m s}^{-1}$ when travelling on a horizontal road. The car experiences a resistance of $kv\,\text{N}$, where $v\,\text{m s}^{-1}$ is the speed of the car and k is a constant. The maximum power of the car's engine is 45 000 W.

   (i)   Show that $k = 50$.   [2]

   (ii) Find the maximum possible acceleration of the car when it is travelling at $20\,\text{m s}^{-1}$ on a horizontal road.   [3]

   (iii) The car climbs a hill, which is inclined at an angle of 10° to the horizontal, at a constant speed of $15\,\text{m s}^{-1}$. Calculate the power of the car's engine.   [3]

5   A particle P of mass $2m$ is moving on a smooth horizontal surface with speed u when it collides directly with a particle Q of mass km whose speed is $3u$ in the opposite direction. As a result of the collision, the directions of motion of both particles are reversed and the speed of P is halved.

   (i)   Find, in terms of u and k, the speed of Q after the collision. Hence write down the range of possible values of k.   [4]

   (ii) Calculate the magnitude of the impulse which Q exerts on P.   [2]

   (iii) Given that $k = \frac{1}{2}$, calculate the coefficient of restitution between P and Q.   [3]
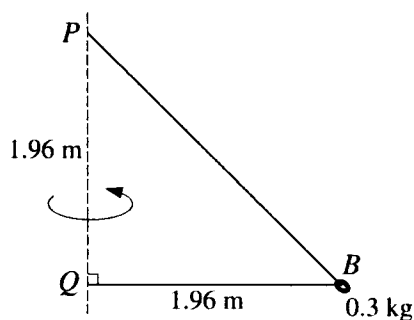
6 **(i)**

Fig. 1

One end of a light inextensible string is attached to a point P. The other end is attached to a point Q, 1.96 m vertically below P. A small smooth bead B, of mass 0.3 kg, is threaded on the string and moves in a horizontal circle with centre Q and radius 1.96 m. B rotates about Q with constant angular speed ω rad s^{-1} (see Fig. 1).

(a) Show that the tension in the string is 4.16 N, correct to 3 significant figures. [2]

(b) Calculate ω. [3]
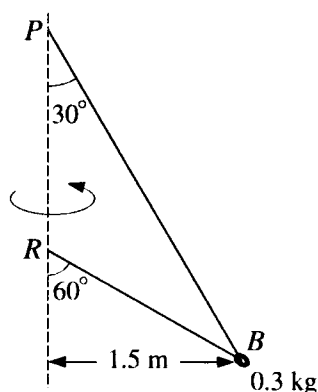
(ii)

Fig. 2

The lower part of the string is now attached to a point R, vertically below P. PB makes an angle $30°$ with the vertical and RB makes an angle $60°$ with the vertical. The bead B now moves in a horizontal circle of radius 1.5 m with constant speed v m s^{-1} (see Fig. 2).

(a) Calculate the tension in the string. [3]

(b) Calculate v. [3]

7 A missile is projected from a point O on horizontal ground with speed 175 m s^{-1} at an angle of elevation θ. The horizontal lower surface of a cloud is 650 m above the ground.

(i) Find the value of θ for which the missile just reaches the cloud. [3]

It is given that $\theta = 55°$.

(ii) Find the length of time for which the missile is above the lower surface of the cloud. [5]

(iii) Find the speed of the missile at the instant it enters the cloud. [4]

8 **(i)** A uniform semicircular lamina has radius 4 cm. Show that the distance from its centre to its centre of mass is 1.70 cm, correct to 3 significant figures. [2]
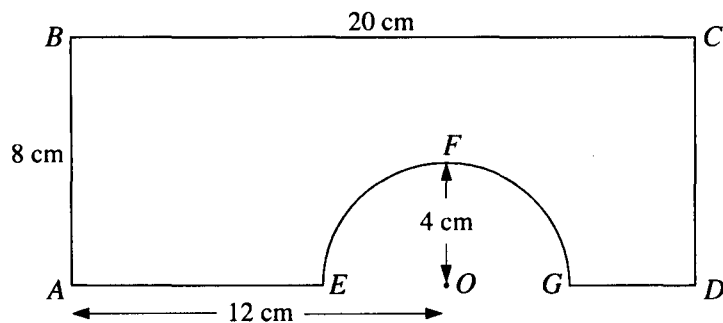
(ii)

Fig. 1

A model bridge is made from a uniform rectangular board, $ABCD$, with a semicircular section, EFG, removed. O is the mid-point of EG. $AB = 8$ cm, $BC = 20$ cm, $AO = 12$ cm and the radius of the semicircle is 4 cm (see Fig. 1).

(a) Show that the distance from AB to the centre of mass of the model is 9.63 cm, correct to 3 significant figures. [5]

(b) Calculate the distance from AD to the centre of mass of the model. [4]
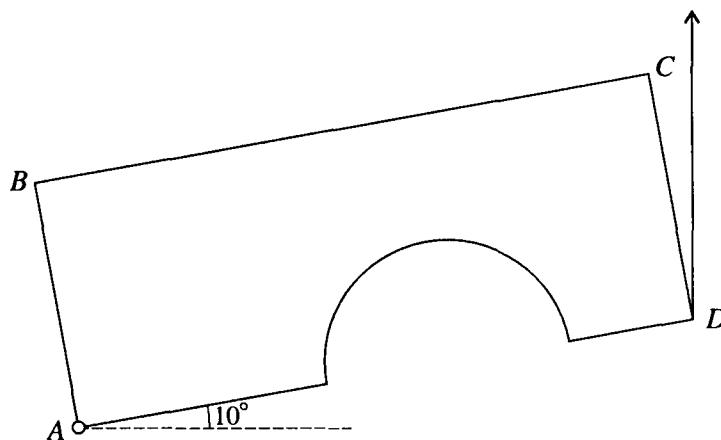
(iii)

Fig. 2

The model bridge is smoothly pivoted at A and is supported in equilibrium by a vertical wire attached to D. The weight of the model is 15 N and AD makes an angle of $10°$ with the horizontal (see Fig. 2). Calculate the tension in the wire. [5]

4729 Mechanics 2

1 (i)	12 x cos55°	M1	
	6.88 m s⁻¹	A1 **2**	
(ii)	12 x cos55° x 0.65	M1	
	(±) 4.47 m s⁻¹ ✓	A1 **2**	✓ 0.65 x their (i) **4**

2	F = 0.2 mg cos30°	M1	=
		A1	= (1.6974m) (49√3/50m)
	0.2mgcos30° x d	B1	a=0.2gcos30°+gsin30°
	mg x d x sin30°	B1	a= (±) 6.60
	d=½x25/(0.2x9.8cos30°+9.8xsin30°)	M1	0 = 5² – 2x6.60d
	1.89 m	A1 **6**	**6**

3	direction of R perp. to wall	B1	
	R at 70° to rod	B1	10° to horiz.
	0.8 x 25cos60° = 1.6 x R sin70°	M1	moments about A
	0.8 x 25 cos60°	A1	
	1.6 x R sin70°	A1	
	R = 6.65 N	A1 **6**	**6**

4 (i)	45 000/v = kv	M1	
	k = 50	A1 **2**	**AG**
(ii)	45 000/20 – 50x20 = 1200a	M1	
		A1	
	a = 1.04 m s⁻²	A1 **3**	
(iii)	P/15 = 50x15 + 1200x9.8sin10°	M1	
		A1	
	41 900 W	A1 **3**	**8**

5 (i)	2mu – 3kmu = –mu + kmv	M1	
	v = …..	M1	attempting to make v the subject
	v = 3u(1 – k)/k	A1	
			3u/k – 3u
	(0 <) k < 1	A1 **4**	not ≤ 1
(ii)	I = mu – – 2mu	M1	or km(3u/k - 3u + 3u)
	3mu	A1 **2**	+ only
(iii)	v = ± 3u	B1	
	e = (u/2 + 3u)/4u	M1	
	e = 7/8 or 0.875	A1 **3**	**9**

6 (i)(a)	T cos 45° = 2.94	M1	Resolving vertically
	T = 4.16 N	A1 **2**	**AG**
(b)	Tcos45° + T = 0.3x1.96ω²	M1	calculates v = 6.81
	(res. horiz.)	A1	(Max 2/3)
	ω = 3.47 rad s⁻¹	A1 **3**	
(ii)(a)	Tcos30° + Tcos60° = 2.94	M1	Resolving vertically
		A1	
	T = 2.15 N	A1 **3**	
(b)	Tcos30° + Tcos60° =0.3v²/1.5	M1	calculates ω = 2.56
	(res. horiz.)	A1	(Max 2/3)
	v = 3.83 m s⁻¹	A1 **3**	**11**

7 (i)	0 = (175sinθ)² – 2x9.8x650	M1	
		A1	
	θ = 40.2°	A1 **3**	
(ii)	Attempt at t₁ , t₂ , t_top or t_total	M1	650 = 175sin55°.t - 4.9t² etc
	5.61, 23.65, 14.63, 29.26	A1	
	t₂ – t₁ or 2(t_top – t₁) or t_total – 2t₁	M1	
		A1	
	time difference = 18.0	A1 **5**	
(iii)	v_h = 175cos55° (100.4)	B1	or KE ½mv²
	v_v = 175sin55° – 9.8 x 5.61	M1	(B1) PE mx9.8x650
	speed = √(88.4² + 100.4²)	M1	v = √(175² – 2x9.8x650)
	134 m s⁻¹	A1 **4**	**12**

8 (i)	(2x4xsinΠ/2)/3xΠ/2	M1	or 4r/3Π
	1.70	A1 **2**	**AG**
(ii)(a)	$\overline{x}$ xd(8x20–Πx4²/2)=10x8x20d–	M1	or 134.9$\overline{x}$ =
	12xΠx4²/2xd		64x4+38.9x12+32x18 (1298.8)
	10x8x20(d) (1600)	A1	64x4
	(8x20–Πx4²/2) (d) (134.9)	A1	38.9x12
	(12xΠx4²/2) (d) (301.6)	A1	32x18
	$\overline{x}$ = 9.63 cm	A1 **5**	**AG**
(ii)(b)	$\overline{y}$ xd(8x20–Πx4²/2)=4x8x20d–	M1	or 64x4=42.7+38.9$\overline{y}$
	1.7xΠx4²/2xd		
	4x8x20 (d)	A1	$\overline{y}$ = 5.49
	1.7d x Π x 4² /2 (13.6Π)	A1M1	135$\overline{y}$ =32x4+38.9x5.49+64x4
	$\overline{y}$ = 4.43 cm	A1 **4**	
(iii)	20cos10° x T	B1	= or
	15cos10° x 9.63	B1	10.6 (A to com)
	15sin10° x 4.43	B1	34.7° ∠ comAH
	20cos10°.T=15cos10°x9.63–	M1	=15x10.6xcos34.7°
	15sin10°x4.43 (needs 3 parts)		
	T = 6.64 N	A1 **5**	**16**